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A STUDY ON PARTIAL DIFFERENTIAL EQUATIONS, DIFFERENTIAL EQUATIONS, AND INTEGRATION IN VEDIC MATHEMATICS

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ABSTRACT

Sūtras and Sub-Sūtras, which are Sanskrit word formulas, are the foundation of Vedic mathematics. The ultimate objective of the scientist is to learn how Vedic mathematics has advanced the field of calculus. As a means of dissecting the applications of Sūtras determinants, matrices, and the discovery of determinants of various networks and situations in Vedic mathematics. With this cycle, it takes extremely little effort to wean students off of minicomputers, regardless of their age or ability level. Improving one's computational abilities is the primary goal of Vedic Mathematics. The consistency of Vedic mathematics is considered by researchers to be its major aspect. Because of this property, mathematics becomes both easy and beautiful to understand. More than that, it sparks innovations.

Keywords: *Partial Differential; Equations; Integration; Vedic Mathematics*

INTRODUCTION

Partial differential equations, differential equations, integration, and other mathematical issues can be better understood and solved by consulting Vedic mathematics, an old Indian body of mathematical knowledge. Vedic mathematics, which has its foundation in Hinduism's holy writings, takes a more comprehensive view of mathematical ideas than is typical in modern mathematical theory. There is a strong relationship between the ancient knowledge contained in the Vedas and the abstract concepts of mathematical analysis, as is shown by studying partial differential equations within the context of Vedic mathematics.[1]

Vedic mathematics offers a unique perspective on differential equations, which are essential in mathematical modelling. Intuitive and non-traditional approaches are prioritised in the Vedic technique in order to reveal the interdependence and velocities of variables. By taking the whole picture into account, practitioners are better able to understand the dynamic nature of mathematical connections and successfully traverse the complex terrain of differential equations. Researchers can learn more about the rules that control change and motion by applying old Vedic wisdom to the study of differential equations [2].

Vedic mathematics also provides a broad perspective on integration, a fundamental concept in calculus. Alternative methods for solving integration issues can be found in the Vedic sutras, which are brief aphorisms that include mathematical ideas [3]. These techniques reveal the symmetry and elegance in mathematical connections while simultaneously improving computing performance. By incorporating Vedic mathematical ideas into integration research, new research directions are revealed and established methods are reevaluated, leading to a more comprehensive comprehension of mathematical harmony and wholeness.[4]

PARTIAL DIFFERENTIAL EQUATIONS IN VEDIC MATHEMATICS

The ancient Indian subcontinental system of mathematics known as Vedic Mathematics is world-renowned for its elegant and efficient solutions to a wide range of mathematical problems. The main areas of study in Vedic Mathematics are algebra and mathematics, but the discipline also has applications in more complex areas of mathematics, such as the solution of partial differential equations (PDEs). Using its distinctive method and the complementary relationship between traditional knowledge and contemporary mathematical theory, this investigation delves into the possible uses of Vedic Mathematics in PDE solution and understanding [5]. The sixteen aphorisms or Sutras that form the basis of Vedic mathematics provide rules on how to solve problems. For example, there is the "Nikhilam Sutra," which literally means "All from 9 and the last from 10." Similar to the idea of boundary conditions in partial differential equations, this sutra discusses complements. In order to learn more about how the solution acts at the domain's boundaries, boundary conditions play an essential role in solving PDEs.[6]

Take the heat equation as an example of a typical sort of partial differential equation (PDE). How does a region's temperature change over time? The heat equation explains it. One possible reading of the Nikhilam Sutra, based on Vedic mathematics, is that it offers guidance on how to properly establish boundary conditions. Vedic mathematics may provide a fresh viewpoint on setting proper conditions at the borders of a physical system by elucidating complements and using them wisely. "Paravartya Yojayet," which translates to "Transpose and Apply," is another sutra that may be relevant when discussing transformations as they pertain to PDE solutions [7]. In order to simplify PDEs and discover novel solutions, transformations are essential. Alternative viewpoints on converting equations may be offered by Vedic Mathematics, which places a focus on mental computations and simplified approaches, perhaps leading to more efficient answers [8].

Using the spatial derivatives in partial differential equations, the "Urdhva-Tiryak Sutra," which translates to "Vertically and Crosswise," may be considered. Calculating a quantity's spatial derivative is a way to understand its relationship to spatial factors. A potential source of inspiration for new approaches to the study and manipulation of spatial derivatives in partial differential equations (PDEs) might be the Vedic technique of vertically and crosswise multiplication. Equations for balancing are an essential part of solving partial differential equations (PDEs), and the "Sunyam Samya Samuccaye," or "Equal additions and subtractions," Sutra offers guidance on this topic. Following the basic idea of conservation rules in physics, which are frequently described by partial differential equations (PDEs), this Sutra stresses the significance of preserving equality while carrying out operations.[10]

The ideas and methods of Vedic mathematics can lead to fresh perspectives on mathematical issues, even while it does not offer direct answers to complicated PDEs. Vedic mathematics, with its emphasis on intuition and holistic analysis, may help us better comprehend the rules that control the behaviour of solutions to partial differential equations (PDEs) [11].

INTEGRATION IN VEDIC MATHEMATICS

A distinct method for resolving mathematical issues is offered by Vedic Mathematics, a set of mathematical procedures and ideas originating from the Vedas, which are ancient Indian scriptures. Its key tenet is the use of simpler procedures and mental computation. One of the cornerstones of calculus, integration, follows the same rules as the rest of Vedic mathematics. It could take a lot of time and complicated algebraic calculations to use the standard way of integration. Vedic mathematics offers many ways of looking at things, which makes things easier and makes your mind more agile. As an example, there is the "Nikhilam Sutra," which literally means "all from 9 and the last from 10." The integration of polynomials is made easier and faster with the help of this sutra.[12]

Think about the integration of a polynomial such as ax^n, where 'a' is a constant and 'n' is a positive integer. According to the Nikhilam Sutra, you should take the final digit out of 10 and the first digit out of 9. A design that makes integration easier is the end product.

As an example, using the Nikhilam Sutra would result in 276 if the coefficient was 723. By using this altered coefficient in the integration process, the complexity of the computations is greatly reduced.[13]

The "Vertically and Crosswise" approach is another useful tool for integration in Vedic mathematics. This approach streamlines integration by making binomial and polynomial multiplication easier. Effectively extending and simplifying equations is made easier with this strategy when working with integrals containing products of polynomials. For integration problems involving quadratic expressions, the "By One More Than the One Before" sutra might be used. By dividing the expression into two pieces, this sutra makes it easier to integrate quadratic polynomials.

"Sub-Sutra," or subsidiary principles that improve the efficiency of major sutras, are also introduced in Vedic mathematics. Among other things, the "Paraavartya Yojayet" Sub-Sutra makes it easier to combine and rearrange words by inverting their order during integration. Vedic mathematics is more than just a set of rules; it encourages mental agility and flexibility, which in turn opens up new avenues of inquiry when addressing problems. Integral concepts, which centre on simplifying expressions and identifying patterns, are congruent with the pattern-oriented, symmetrical, and simple Vedic mathematics [14].

INTRODUCTION TO DIFFERENTIAL CALCULUS

Achieving the development of integral calculus and differential calculus is the pinnacle of mathematical achievement. Differential calculus is invaluable in many practical contexts where determining the relationship between two parameters is essential, such as [15]

By sketching an angle at a specific location and then calculating the slope, the geometric derivative of any function at that position may be found. Given that y equals f(x), then:

$$\frac{dy}{dx} = f'(x)$$

By using Dhvaja Ghata (power)

When solving the quadratic equation ax2 + b x + c, the first differential of each term may be found by multiplying its DhvajaGhata (power) by its Anka (coefficient) and then dividing by one.

Example 1:

The quadratic formula x2 - 9x + 14 has a root. Get its derivative.

Let E = x2-9x + 14

Derivative of y with respect to x, in accordance with the present approach

$$\frac{dy}{dx} = f'(x) = \frac{d}{dx}(x^2) - 9\,\frac{d}{dx}(x) + \frac{d}{dx}(14)$$

$$= 2x - 9(1) + 0$$

$$\frac{dy}{dx} = 2 = 2x - 9$$

Finding the first differential of each term in the quadratic formula x2 - 9x + 14 using Dhvaja Ghata, we get 2x for x2, -9x for x, and zero for 14 as a result.[16]

Therefore

$$D1 = f\frac{d}{dx}(x^2 - 9x + 14) = 2x - 9.$$

Calana-Kalanābhyām Sūtra: Meaning

Calculus of Differentials This Sūtra displays the discriminant of the quadratic first differential as well as its square root. This Sūtra was recognised by ŚRĪ BHĀRATĪ KRA as the formula for obtaining the two roots of a given quadratic equation in calculus. He contends that the square root of the discriminant of the original quadratic equation is equal to the first differential. As a result, two simple equations may be used to find the roots of specified quadratic equations [17].

Example 2:

Complete the square root of the equation x^2 - x - 12 = 0.

If we take the first differential, D1, which is equal to 2x minus 1, then

The square root of the discriminant is $\pm\sqrt{1+48} = \pm\sqrt{49} = \pm 7$

As per above rule, D1 =± √Discriminant ∴ 2x - 1 = ±7

Consequently, the original equation may be simplified into two equations:

∴ 2x - 1 = ±7

∴ 2x = ±7 + 1

∴ x = 4 OR x = −3

Sūtra in solving successive differentiation both vertically and crosswise

Multiplication of two functions by crosswise Sūtra as a derivative

For example, if y = u. w and both u and w are variables x, then

$$\frac{dy}{dx} = u.\frac{dw}{dx} + w.\frac{du}{dx} \ldots\ldots [P]$$

Vedic Crosswise Sūtra may be used to determine the differentiation of multiplication in this sort of connection, provided that one is familiar with the standard formula.[18]

Example 3:

Find $\frac{dy}{dx}$ $if\ y = x^2.2^x$

Using the existing procedure and the aforementioned formula [P],

u = x2 & w = 2x ;

Using the Vedic Approach

The derivatives of u and w may be calculated using the conventional formula, By using the Vedic Crosswise Sūtra

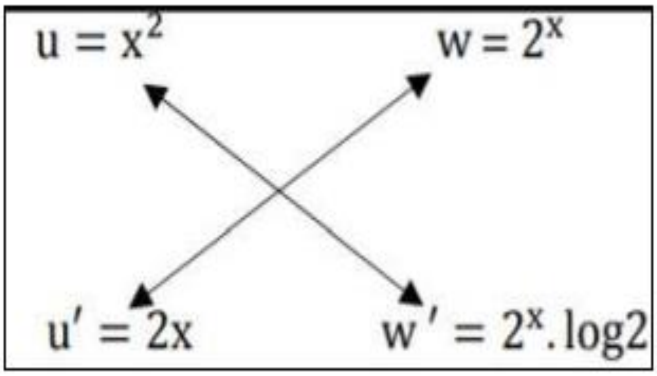

Figure 1: From the above figure

Example 4:

$$\frac{dy}{dx} = x^2(2^x \log 2) + 2x(2^x$$

Using the present technique,

$$u = (2x^2 + 3x + 7) \; w = (5x^2 + 7)$$

$$\frac{dy}{dx} \; if \; y = (2x^2 + 3x + 7)(5x^2 + 7)$$

$$u' = \frac{du}{dx} = 4x + 3 \quad \& \; w' = \frac{dv}{dx} = 10x$$

$$\therefore \frac{dy}{dx} = (2x^2 + 3x + 7)10x + (5x^2 + 7)(4x + 3) \qquad \because \text{from}[P]$$

By using Vedic Method

To achieve this, Dhvaja Ghata,

$$u' = \frac{du}{dx} = \frac{d}{dx}(2x^2 + 3x + 7)$$

$$\therefore u' = 4x + 3 \; \& \; w' = \frac{dw}{dx} = \frac{d}{dx}(5x^2 + 7)$$

$$\therefore w' = 10x$$

By using the Vedic Crosswise Sūtra

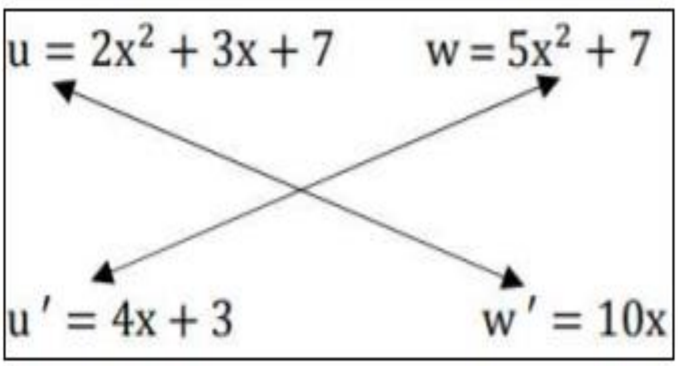

Figure 2: Vedic Crosswise Sūtra

$$\therefore \frac{dy}{dx} = (2x^2 + 3x + 7)10x + (5x^2 + 7)(4x + 3)$$

Regression analysis of the polynomial ratio

A polynomial's derivative when divided by another

It is easy to compute the vertically and crosswise Sūtra derivative of the division of two polynomial functions if u and w are both polynomials.[19]

Example 5:

Differentiate

$$y = \frac{2 + 4x}{2x + 2x^2}$$

The division rule is utilised,

$$\frac{dy}{dx} = \frac{(2x + 2x^2)4 - (2 + 4x)(2 + 4x)}{[2x + 2x^2]^2}$$

$$= \frac{(8x + 8x^2) - (4 + 16x + 16x^2)}{[2x + 2x^2]^2}$$

$$= \frac{8x + 8x^2 - 4 - 16x - 16x^2}{[2x + 2x^2]^2}$$

$$\therefore \frac{dy}{dx} = \frac{-4 - 8x - 8x^2}{[2x + 2x^2]^2}$$

The sūtra that was stated before is fairly extensive, however it is necessary to get the derivative of division of two polynomials by utilising crosswise division [20]. The numerator of this solution is readily apparent from the picture below, and the denominator is the square of the term in the denominator.

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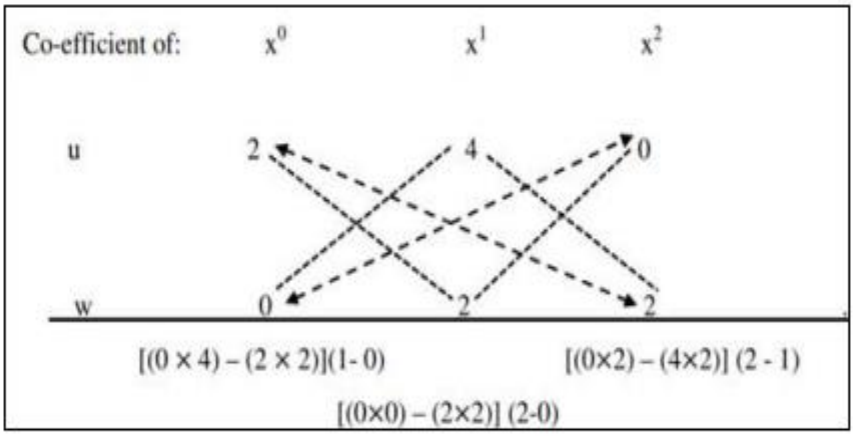

Figure 2: Co-efficient

Let:

$$y = \frac{2 + 4x}{2x + 2x^2} = \frac{2x^0 + 4x + 0x^2}{0x^0 + 2x + 2x^2}$$

$$\frac{dy}{dx} = \frac{(0 \times 4 - 2 \times 2)(1 - 0) + (0 - 2 \times 2)(2 - 0)x + (2 \times 0 - 4 \times 2)(2 - 1)x^2}{[2x + 2x^2]^2}$$

$$= \frac{(-4) + (-4)2x + (-8)x^2}{[2x + 2x^2]^2}$$

$$\therefore \ \frac{dy}{dx} = \frac{-4 - 8x - 8x^2}{[2x + 2x^2]^2}$$

Integration based on partial fraction by parāvartya yojayet sutra

Suppose f(x) and h(x) are two polynomials. The rationality of the function is determined by the range of values that its denominator can take [21]. In this case, since h(x) has a non-zero value, the function is rational and proper since its degree is greater than f(x). You may represent it as a partial fraction using the following table, where L, M, N, and O are real values.[22]

Table 1: Integration based on partial fraction

Rational Form	Partial Form
$\frac{Px^2 + qx + r}{(x - a)(x - b)(x - c)}$	$\frac{L}{(x - a)} + \frac{M}{(x - b)} + \frac{N}{(x - c)}$

$\frac{Px^2 + qx + r}{(x-a)^2 (x-b)}$	$\frac{L}{(x-a)} + \frac{M}{(x-a)^2} + \frac{N}{(x-b)}$
$\frac{Px^2 + qx + r}{(x-a)^3 (x-b)}$	$\frac{L}{(x-a)} + \frac{M}{(x-a)^2} + \frac{N}{(x-a)^3} + \frac{0}{(x-b)}$

Dividing the denominator into irreducible components is what partial fractions are all about.

Integration by parts

When one function can be easily distinguished (u) and the other can be integrated (w), the integrand is stated as a product of the two functions, and integration by parts is utilised. Consider the following expressions for w and u [23].

$$\int u.w\,dx = u\int w dx - \int \left\{\frac{d}{dx}(u)\int w\,dx\right\} dx$$

In order to integrate a function that is the product of two functions, where the integral of the first function is known and the second function is integrated, one must exercise caution while selecting the first and second functions.

When the term "LIATE" appears first, we have another option: the first function, which is the L-logarithmic function. For A-algebraic Inverse Vectors Three-dimensional geometry and exponential.

Integration of the product of two functions by vertically and crosswise

The following formula, which involves multiplying the functions (of x) crosswise and adding them with an alternative sign, can be used to solve the integration of the product of two functions.[24]

$$\int u.w\,dx = uw' - u_1w'' + u_2w''' - u_3w'''' + u_4w''''' - \ldots\ldots\ldots\ldots$$

Example 6:

Discover the integrals that follow

Since the present approach

Let:

$$I = \int x^2 e^{2x} dx$$

Let's use the integration by parts formula to find $u=x^2$ and $w=e^{2x}$,

$$I = x^2 \int e^{2x}\,dx - \int \left(\frac{d}{dx}(x^2) \int e^{2x}\,dx\right) dx$$

$$= x^2 . \frac{e^{2x}}{2} - \int \left[(2x)\left(\frac{e^{2x}}{2}\right)\right] dx = \frac{x^2 e^{2x}}{2} - \int x\, e^{2x}\,dx$$

Once more, the integration by parts approach was used for $\int x^2 e^{2x} dx$

$$= \frac{1}{2} x^2 e^{2x} - \left[x \int e^{2x}dx - \int \left(\frac{d}{dx}(x) \int e^{2x}dx\right) dx\right]$$

$$= \frac{1}{2} x^2 e^{2x} \left[x \int e^{2x}dx - \int \left[(1)\left(\frac{e^{2x}}{2}\right)\right] dx\right]$$

$$= \frac{1}{2} x^2 e^{2x} - \left[\frac{1}{2}\, x\, e^{2x} - \frac{1}{2} \int e^{2x}\,dx\right]$$

$$= \frac{1}{2} x^2 e^{2x} - \left[\frac{1}{2}\, x\, e^{2x} - \frac{1}{2} \left[\frac{e^{2x}}{2}\right]\right] + c$$

$$= \frac{1}{2} x^2 e^{2x} - \left[\frac{1}{2}\, x\, e^{2x} - \frac{1}{4}\, e^{2x}\right] + c$$

$$\therefore I = \frac{1}{2} x^2 e^{2x} - \frac{1}{2}\, x\, e^{2x} + \frac{1}{4}\, e^{2x} + c$$

Vertically and crosswise Sūtra:

Let:

$I = \int x^2 e^{2x} d$

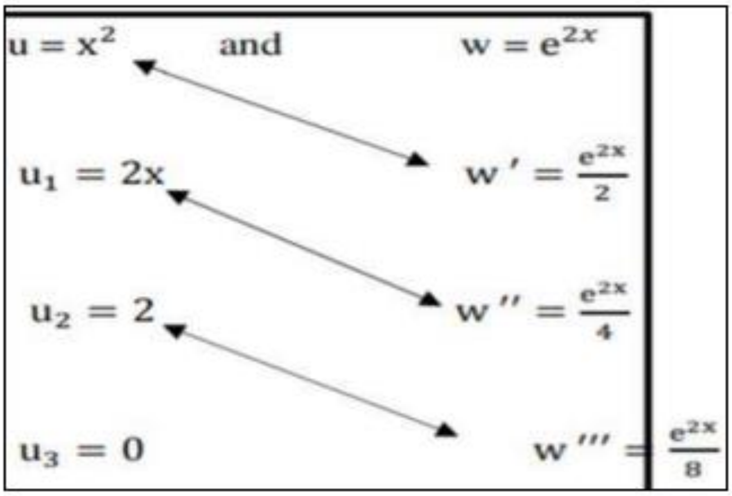

Figure 4: Vertically & Crosswise Sūtra

Using the formula

$$\therefore \int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + c$$

CONCLUSION

Complex and comprehensive computations may be handled with more precision and less time spent using Vedic Sūtras compared to estimations based on regular mathematics. Virtual reality also improves memory and cognitive sharpness. Consistency is the key of Vedic mathematics. It creates a relaxing and pleasant atmosphere because of its feature. It inspires progress. In the Vedic system, the wonderful logic between polynomial maths and number-crunching is plain to see. Vedic calculations based on texts such as the Ūrdhva-Tiryagbhyām Sūtra, Nikhila Sūtra ṁ, and Ānurūpye a Sub-ṇ Sūtra can be utilised for the purpose of planning exceptionally fast events. Digital Signal Processors/Vedic Multipliers/Reconfigurable Fast Fourier Transform.

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